The Hasse-Witt invariant of generalized Fermat Curves

Herivelto Borges (hborges@icmc.usp.br)
USP

Abstract. (Joint work with C. Gonçalves)

For any irreducible algebraic curve $\mathcal{X}$ defined over a field $\mathbb{K}$, its genus $g(\mathcal{X})$ is certainly the most famous birational invariant. If $\mathbb{K}$ has characteristic $p > 0$, then the curve $\mathcal{X}$ has another important birational invariant, called Hasse-Witt invariant or the p-rank of $\mathcal{X}$, which can be defined as the integer $\gamma := \gamma(\mathcal{X}) \in \{0, 1, \ldots, g(\mathcal{X})\}$ for which

$$Pic_0\mathbb{K}(\mathcal{X}, p) = (\mathbb{Z}_p)^\gamma.$$

It is well known that number $\gamma(\mathcal{X})$ is closely related to certain arithmetic and geometric properties of $\mathcal{X}$. For instance, $\mathbb{F}_{p^{2n}}$-maximal and $\mathbb{F}_{p^{2n}}$-minimal curves $\mathcal{X}$ have $\gamma(\mathcal{X}) = 0$, and curves with large p-rank have a somewhat small automorphism group. In this talk, we discuss recent results on the computation of the p-rank of generalized Fermat curves

$$\mathcal{F}_{m,n} : y^m + x^n + 1 = 0,$$

and some of its consequences. In particular, via Deuring-Shafarevich formula, we will present the p-rank of some other important classes of curves such as the Dickson-Guralnick-Zieve curves.